

**PART I. Respond to each of the nine statements in this part by telling whether it is TRUE, FALSE, or UNCERTAIN, and provide a brief justification for your answer.**

1. If a consumer's preferences display Diminishing Marginal Rate of Substitution, then any utility function that represents those preferences must be concave.
2. If  $Y$  is a net complement for  $X$ , but it is a gross substitute for  $X$ , then  $Y$  must be a Giffen good.
3. A firm with the production function  $q = f(K, L) = 10KL / (K + L)$  has the elasticity of factor substitution equal to  $1/2$ .
4. If  $x$  is a luxury good and income is redistributed from the rich to the poor, we would expect the price of  $x$  to rise.
5. For a long-run equilibrium in a competitive industry to exist, the firms must have constant returns to scale.
6. Suppose a per unit tax,  $t$  dollars, is imposed on the output of a monopoly which has constant cost of production and is facing a market demand  $q = p^{-e}$ , with  $e > 1$ . Then the monopoly price will be raised by less than  $t$  dollars.
7. If a monopolist produces a single good at a constant marginal cost  $c > 0$  and faces the demand curve,

$$D(p) = \begin{cases} 0 & \text{if } p > 20 \\ 10/p & \text{if } p \leq 20 \end{cases}$$

then the monopolist maximizes profit by producing the largest possible output level.

8. Factor demand curves for a competitive firm using a decreasing-returns-to-scale technology are always downward-sloping with respect to the factor's own price.
9. In a 2-output economy, if both production technologies involve decreasing returns to scale, then the economy's Production Possibilities Frontier will be concave (that is, the Production Possibilities Set will be a convex set).

**PART II. Please choose ONE of the following three questions to answer.**

10. Consider a firm producing a single commodity at two different plants. The total production is  $x$ , and the respective production quantities at the two plants are  $x_1$  and  $x_2$ . Suppose that the production cost functions at the two plants are:
- $$C_1 = \gamma (x_1)^2 \text{ where } \gamma > 0,$$
- and  $C_2 = (x_2)^2$ ,
- and assume that transportation costs are zero.
- Suppose the firm wants to produce 100 units of the commodity at minimal cost. Find the first-order necessary conditions for the solution to this problem, and verify that these are also sufficient conditions.
  - What are the cost-minimizing output levels for each plant, if the total is to be 100?
  - Generalize the above results by deriving general functions for plant quantities  $x_1^*$  and  $x_2^*$ , as functions of the total production target  $x$ .
  - Supposing the firm is a price-taker and the market price is  $p$ , what will be the plant-specific supply functions, as functions of  $p$ ?
11. In a two-person (A and B) two-commodity (X and Y) exchange model, the utility functions are given as:  $U_A = X_A + 2Y_A$  and  $U_B = X_B Y_B$ . Initially, A has one unit of X and  $\frac{1}{2}$  unit of Y and B has only  $\frac{1}{2}$  unit of Y and no X.
- Find the offer curve for each individual and the contract curve. Determine the efficient distribution of X and Y, using Y as the numeraire good with  $P_Y = 1$ .
  - Next, assume that the government wishes to redistribute income so that A would have  $\frac{1}{4}$  unit of X and  $\frac{5}{8}$  unit of Y, and B would have  $\frac{3}{4}$  unit of X and  $\frac{3}{8}$  unit of Y. What would be the size of the lump-sum transfer? Would this new distribution be still on the contract curve?
  - Explain and illustrate your results in an Edgeworth Box diagram.
12. Suppose a consumer's preferences are described by the utility function  $u = \ln x + m$ , where  $x$  is particular consumption good (sold at price  $p$ ), and  $m$  is money spent on all other goods, so its price is 1.
- Derive the consumer's expenditure function  $e(p,u)$ .
  - Is this expenditure function non-decreasing in  $p$ ? Is it concave in  $p$ ?
  - Suppose that, initially,  $p = 1$  and  $u = 99$ , and then  $p$  rises to  $p = 2$ . What is the compensating variation ( $cv$ ) measure of welfare change, associated with this price rise?
  - What is the equivalent variation ( $ev$ ) for the same price rise? Explain why it is or is not equal to  $cv$ .

**PART III. Please choose ONE of the following two questions to answer.**

13. Suppose that Ralph can consume either leisure ( $l$ ) or Other Goods ( $x$ ). Other Goods are measured in dollars, so the price of Other Goods is \$1. Ralph has 5000 hours of time available to him each year, and can divide the 5000 hours between leisure and work. Suppose that he makes \$8 an hour in his job, and that his utility function is  $U(l,x) = l^{0.6}x^{0.4}$ .
- How much of  $x$  and  $l$  will he consume in a year?
  - Suppose that a new governmental income redistribution program is implemented in Ralph's country. Specifically, the government gives a person 50¢ for each dollar Ralph earns up to \$15,000. Between \$15,000 and \$25,000 earned, Ralph gets no extra money from the government. Between \$25,000 and \$40,000 earned Ralph must give back 50¢ for every \$1 he earns.
    - In order to gain insight into the following question, identify Ralph's annual budget constraints in parts (a.) and parts (b.).
    - Some people have argued that such a governmental program will *never* induce an increase in hours worked. Critique their argument and discuss Ralph's likely response to the program. (Note that for this subpart you do not have to solve for the specific amount of  $x$  and  $l$  Ralph will consume.)
14. Suppose a market has the inverse demand curve  $P = a - bQ$  and is currently served by two Cournot-style duopolists that both have the same total cost function, namely,  $C_i = aq_i$ , where  $C_i$  is firm  $i$ 's total cost and  $q_i$  is firm  $i$ 's production, for  $i = 1,2$ .
- A third firm is a potential entrant in this market, but it has production costs that are double those of the two incumbents. That is,  $C_3 = 2aq_3$ . Use formal modeling to answer the following questions:
- Assuming Cournot-type behavior, can this third firm profitably enter the market?
  - Would the entry of this high-cost firm improve the social efficiency of the outcome? Explain.